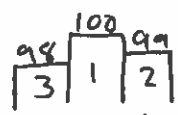
Solutions

4.2: Selections and Arrangements

Question 1. There are 100 racers in a race. The first three will stand on the podium at the end. How many different possible orderings of the podium are there?

$100 \cdot 99 \cdot 98 = 970{,}200$ orderings

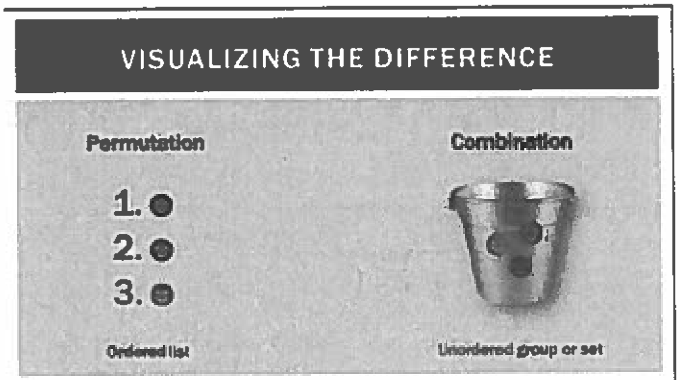


Question 2. There are 100 racers in a race. The first three will stand on the podium at the end. A photographer will take a picture of this podium. How many different collections of people are possible to be in this picture?

$$\frac{100 \cdot 99 \cdot 98}{3 \cdot 2 \cdot 1} = 161{,}700$$

Permutations and Combinations.

A permutation of n items taken r at a time is an *ordered* list of r items chosen from n.
A combination of n items taken r at a time is an *unordered* set of r items chosen from n.



The Arrangement Principle. (Permutations) The number of ways to form an ordered list of r distinct elements drawn from a set of n elements is

$$P(n,r) = n \cdot (n-1) \cdot \cdots \cdot (n-r+1) = \frac{n!}{(n-r)!}.$$

Example 1. A baseball team has a 24-man roster. How many different ways are there to chose a 9-man batting order?

$$P(24,9) = \frac{24!}{15!} = 474{,}467{,}051{,}520$$

Example 2. How many different ways are there to rearrange the letters in the word GOURMAND?

$$P(8,8) = \frac{8!}{0!} = 8! = 40{,}320$$

Example 3. A kitchen drawer has 10 different plastic food containers and 10 different lids, but any lid will fit on any container. How many different ways are there to pair up containers with lids?

$$P(10,10) = \frac{10!}{0!} = 10! = 3{,}628{,}800$$

Example 4. An urn contains 10 ping-pong balls, numbers 1 through 10. Four balls are drawn from the urn in sequence., and the numbers on the balls are recorded. How many ways are there to do this, if

(a) the balls are replaced before the next one is drawn.

(b) the balls are drawn and not replaced.

(a) $10^4 = 10{,}000$

(b) $P(10,4) = \frac{10!}{6!} = 5{,}040$

The Selection Principle. (Combinations) The number of ways to choose a subset of r elements from a set of n elements is

$$C(n,r) = \binom{n}{r} = \frac{n!}{r!(n-r)!}.$$

Example 5. A baseball team has a 24-man roster. How many different ways are there to choose a group of nine layers to start the game?

$$C(24,9) = \frac{24!}{9!15!} = 1{,}307{,}504$$

Example 6. Suppose an urn contains 10 ping-pong balls numbered 1 through 10. Instead of drawing four balls in sequence, reach in and grab a handful of four balls. How many different handfuls can you grab?

$$C(10,4) = \frac{10!}{6!4!} = 210$$

Example 7. How many ways are there to rearrange the letters in the word PFFPPPPFFFF?

$$C(10,4)\cdot C(6,6) = \frac{10!}{6!4!}\cdot\frac{6!}{0!6!} = 210\cdot 1 = 210$$

Example 8. How many solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 + x_5 = 13,$$

if $x_1, \ldots, x_5$ must be non-negative integers?

$$||+|||+|||||+|||+$$

$$C(17,4) = \frac{17!}{4!13!} = 2380$$

The Binomial Theorem.

Theorem 1. *Let j and k be non-negative integers such that $j + k = n$. The coefficient of the $a^j b^k$ term in the expansion of $(a+b)^n$ is $C(n,j) = C(n,k)$.*

No Homework.
Practice Problems. Section 4.2: 1-13, 8-20, **29-30**